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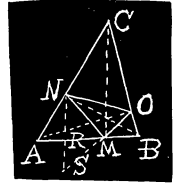
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I. Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Assuming the points N and O in the sides AC and BC , $NM+MO$ is a minimum, if $\angle NMA = \angle OMB$; for letting fall the perpendicular NR and extending it to S by its own length, OMS becomes a straight line. It follows from this that in the case of MNO being a triangle of minimum perimeter, $\angle NMA = \angle BMO = \alpha$, $\angle ANM = \angle CNO = \beta$, $\angle MOB = \angle NOC = \gamma$.



Now, $\alpha + \beta = 180^\circ - A$, $\alpha + \gamma = 180^\circ - B$, $\beta + \gamma = 180^\circ - C$.
 $\therefore \alpha + \beta + \gamma = \frac{1}{2}(540^\circ - 180^\circ) = 180^\circ$. $\therefore \alpha = C$, $\beta = B$, $\gamma = A$.
 Consequently, the triangle MNO is the pedal triangle.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

When we wish to find a point P on a given line so that the sum of the distances $PR+PS$ to two given points, is a minimum, it is easy to show that PS and PR must make equal angles with the given line. Let M, L, K be the feet of the altitudes; M on AB , L on AC , K on BC . Then if M, L are fixed, K is the point on BC such that $KL+KM$ is a minimum, since KL, KM make equal angles with BC . Similarly, for M, K and L, K fixed in turn, respectively. $LM+LK$ is a minimum and $ML+MK$ is a minimum for the reason cited above.

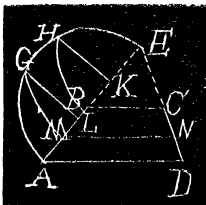
This seems the easiest and simplest proof.

340. Proposed by J. H. MEYERS, S. J., Sacred Heart College, Augusta, Ga.

Given trapezoid $ABCD$. Prolong AB and CD , the non-parallel sides, to meet in E . On AE as diameter construct semi-circle $AHGE$. With BE as radius construct arc BG . Draw GK perpendicular to AE . Bisect AH at L . Erect KH perpendicular to AE . Construct arc HM with HE as radius. Draw MN perpendicular to DC . Prove that MN bisects the trapezoid $ABCD$, angles ADC and BCD being right angles.*

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

We may generalize this theorem by drawing the line MN parallel to AD , instead of perpendicular to DC .



$$\triangle ADE : \triangle EMN = AE^2 : EM^2 = AE^2 : EH^2 = AE^2 : AE \times EL = AE : EL.$$

$$\therefore \triangle ADE - \triangle EMN : \triangle EMN = AE - EL, \text{ or}$$

$$AMND : \triangle EMN = AL : EL \dots (I).$$

$$\triangle EMN : \triangle EBC = ME^2 : BE^2 = KE^2 : GE^2 = AE \times EL : AE \times EK = EL : EK.$$

$$\therefore \triangle EMN - \triangle EBC : \triangle EMN = EL - EK : EL, \text{ or}$$

$$MBCN : \triangle EMN = LK : EL \dots (II).$$

$$\text{Comparing (I) and (II), } AMND : MBCN = AL : LK = 1 : 1.$$

*The reading of this problem has been slightly changed to correspond to the figure. ED. F.

$$\therefore AMND = MBCN.$$

Note. If the angles at C and D are right, MN , of course, is perpendicular to CD . As the proposer states this theorem, it is too restricted.

Also solved by G. B. M. Zerr, J. A. Caparo, A. H. Holmes, and analytically by V. M. Spunar.

341. Proposed by FRANK LOXLEY GRIFFIN, S. M., Ph. D., Instructor in Mathematics, Williams College.

Given $\rho = \cos(m/n)\theta$, where m and n are integers without a common factor. Deduce rules for finding by inspection:

- (1) The angle between the beginning and end of any loop of this curve;
- (2) The number of distinct loops. [A loop is a portion of the curve between consecutive zero radii vectores.]

Solution by the PROPOSER.

- (1) Consecutive values of $\frac{m}{n}\theta$ which make ρ vanish must differ by π .

Hence, consecutive values of θ differ by $n\pi/m$.

- (2) Since ρ is a periodic function of θ , the period being $\frac{2n\pi}{m}$, the rectangular coordinates have the period $2n\pi$. The whole figure being repeated with that period, attention may be confined to the interval $\theta = \frac{n\pi}{m} \frac{\pi}{2}$ to $\theta = \frac{n\pi}{m} \frac{\pi}{2} + 2n\pi$. In this interval there are $2m$ loops, as follows from (1). It remains to be seen when these loops will be all distinct.

The coincidence of two loops occurs only when a loop in which ρ is negative repeats one in which ρ is positive. In that case, any point (ρ_1, θ_1) on one loop coincides with $[-\rho_1, \theta_1 + \pi(2\lambda + 1)]$ on the other [λ , some integer]. But, since $-\rho_1 = a \cos \frac{m}{n} [\theta_1 + \pi(2\lambda + 1)]$, and $-\rho_1 = -a \cos \frac{m}{n} \theta_1$, it is clear that $\frac{m}{n}(2\lambda + 1)\pi$ is an odd multiple of π , say $\frac{m}{n}(2\lambda + 1) = 2k + 1$. This is impossible if either m or n is even. Moreover, if both are odd, λ and k can always be found to satisfy the condition; e. g., $\lambda = \frac{n-1}{2}$, $k = \frac{m-1}{2}$. Each loop for $\rho < 0$ will then repeat a loop for $\rho > 0$. Hence the number of distinct loops is (a) $2m$ if either m or n is even, (b) m if both m and n are odd.

Remark. The curve $\rho = \sin \frac{m}{n}\theta$ is simply the foregoing curve rotated through the angle $-\frac{n\pi}{m} \frac{\pi}{2}$. Hence the conclusions reached above apply equally to this curve.

Also solved by G. B. M. Zerr, V. M. Spunar, and J. W. Clawson, a Sophomore in Williams College.